

# Optimal choice of trapezoidal shaping parameters in digital nuclear spectrometer system

ZHANG Huaiqiang<sup>1,2,\*</sup> GE Liangquan<sup>3</sup> TANG Bin<sup>1</sup> LIU Tingli<sup>2</sup>

<sup>1</sup>Engineering Research Center of Nuclear Technology Application (East China Institute of Technology), Ministry of Education, Nanchang 330013, China

<sup>2</sup>School of Nuclear Engineering and Geophysics, East China Institute of Technology, Nanchang 330013, China

<sup>3</sup>School of Information Science and Technology, Chengdu University of Technology, Chengdu 610059, China

**Abstract** Trapezoidal shaping method is widely applied to pulse amplitude extraction in digital nuclear spectrometer system, the optimal selection of the shaping parameters can improve the energy resolution and pulse counting rate. From the view of noise characteristics, ballistic deficit compensation characteristics and pulse pile-up characteristics, in this paper the optimal selection of the trapezoidal shaping parameters is studied on. According to the theoretical analysis and experimental verification, the optimal choice of trapezoidal shaping parameters is similar to the triangle, the rise time is longer and the flat-top width is shorter.

**Key words** Trapezoidal shaping parameter, Energy resolution, Ballistic deficit, Pulse counting rate, Digital nuclear spectrometer

## 1 Introduction

Digital processing of nuclear signal has been 40 years of history, as early as in 1973, since Koeman *et al.* used digital filter to process the nuclear signal in Philip laboratory<sup>[1,2]</sup>. At the same time, the digital filter was designed, which was used to change the nuclear signal into trapezoidal pulse, and a set of X-ray energy spectrum measuring system was established based on this digital filter<sup>[3]</sup>. Due to the limitation of hardware level at that time, the ADC of the key device was 5 Bits and 2.5 MHz, the digital measuring system failed to go beyond the analog system in performance etc.<sup>[4,5]</sup>. The trapezoidal filter algorithm has the advantages in ballistic deficit compensation, energy resolution, pulse throughput, which is often used to process the digital nuclear signal worldwide. For example, the companies of Canberra, XIA and ORTEC have used the trapezoidal shaping filter into the series of products of the digital nuclear spectrometer<sup>[6]</sup>. The trapezoidal shaping algorithm has been discussed by the relevant

domestic university and research institutes in China, which mainly included the realization and improvement of trapezoidal shaper<sup>[7-11]</sup>, but hardly in the research on the choice of optimal forming parameters of the trapezoidal filter. In this paper, starting from the performance of ballistic deficit compensation, noise filtering and pulse pile-up, after comparing the energy resolution and pulse counting rate in different trapezoidal filter shaping parameters, the optimization of trapezoidal shaping parameters is studied in order to improve and strengthen the trapezoidal shaping algorithm which is further applied in digital nuclear signal processing system.

## 2 Trapezoidal shaping algorithm

The input exponential signal is shaped into isosceles trapezoid, which trapezoidal rise time and width can be adjusted, trapezoidal expression in the time domain is described as Eq.(1)<sup>[12]</sup>.

$$V_o = \sum_{i=1}^4 y_i(t) \quad (1)$$

Supported by National High Technology Research and Development Program of China (Nos.2012AA061804 and 2012AA061803), East China Institute of Technology Science Foundation (No.DHBK201111) and Open-ended Foundation (No.HJSJYB2011-18) from the Chinese Engineering Research Center

\* Corresponding author. E-mail address: zhanghq821@163.com

Received date: 2013-05-06

where the parameter  $V_o$  is the trapezoidal shaping output,  $y_i(t)$  is the four sides of the trapezoid, they are respectively as shown in Fig.1.

$$\begin{aligned} y_1 &= Atu(t)/t_a \\ y_2 &= -A(t-t_a)u(t-t_a)/t_a \\ y_3 &= -A(t-t_b)u(t-t_b)/t_a \\ y_4 &= A(t-t_c)u(t-t_c)/t_a \end{aligned}$$

From Fig.1, the  $A$  is the signal amplitude, the  $t_a$  and  $D$  are respectively rise time and flat-top width of trapezoid,  $t_b=t_a+D$ ,  $t_c=t_a+t_b$ , the  $t_c$  is total width of trapezoid, the  $\mu(t)$  is step function, let  $T_s$  be a sampling frequency,  $n_a=t_a/T_s$ ,  $n_b=t_b/T_s$ ,  $n_c=t_c/T_s$ , Eq.(1) is converted into Eq.(2) by the Z transform.

$$V_o(z) = \frac{Az(1-z^{-n_a} - z^{-n_b} + z^{-n_c})}{n_a(z-1)^2} \quad (2)$$

$$\begin{aligned} H(z) &= \frac{V_o(z)}{V_i(z)} \\ &= \frac{(z^{-1} - z^{-(n_a+1)} - z^{-(n_b+1)} + z^{-(n_c+1)}) - a(z^{-2} - z^{-(n_a+2)} - z^{-(n_b+2)} + z^{-(n_c+2)})}{n_a(1-2z^{-1} + z^{-2})} = \frac{z^{-1}(1-az^{-1})(1-z^{-n_a} - z^{-n_b} + z^{-n_c})}{n_a(1-z^{-1})^2} \quad (3) \end{aligned}$$

### 3 Optimal choice of trapezoidal shaping parameters

Optimal choice of trapezoidal shaping parameters needs to weigh the characteristics of noise filtering,

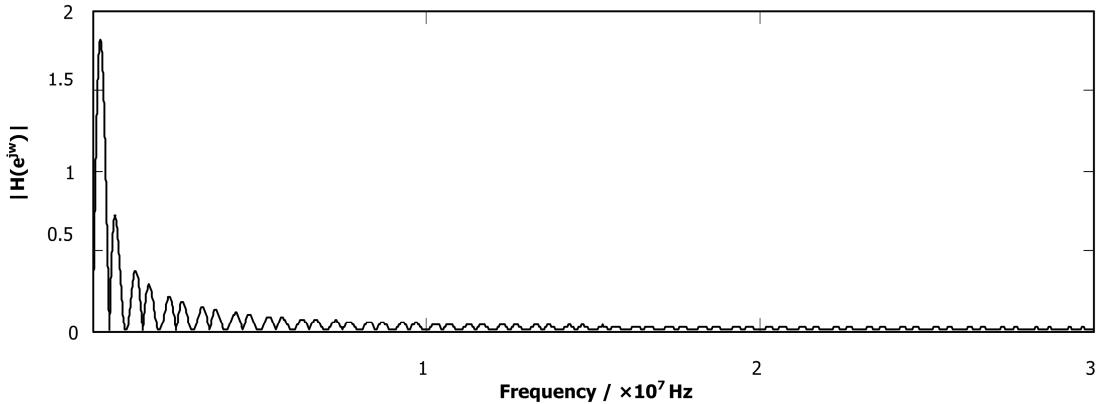


Fig.2 Amplitude-frequency response curve of trapezoidal shaper.

#### 3.1 Noise characteristics of trapezoidal shaper

Trapezoidal shaping algorithm is focused on the characteristics in time domain, but it is equivalent to the filtering operation in the frequency domain, and the result of signal filtering is important to subsequent pulse amplitude analysis, so it is necessary to further

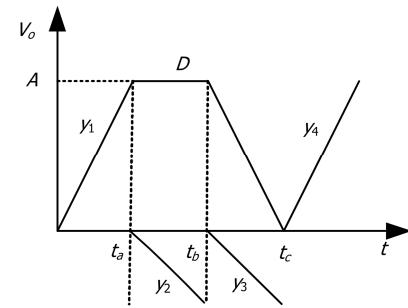


Fig.1 Diagram of trapezoidal shape.

The expression of single exponential function is  $V_i(t)=Ae^{-t/\tau}$ , where the parameter of  $A$  is the signal amplitude and  $\tau$  is the time constant, after Z transform, which is described as  $V_i(z)=Az/(z-a)$ , let  $a=\exp(-T_s/\tau)$ , the transfer function of single exponential signal is shown as Eq.(3) in trapezoidal shaping process<sup>[13]</sup>

ballistic deficit, pulse pile-up and amplitude extraction, in order to guarantee the maximal throughput and the best energy resolution in the digital nuclear spectrometer system.

explore the frequency characteristics of the algorithm. Eq.(1) and the input signal of  $V_i(t)=Ae^{-t/\tau}$  are processed by Fourier transform, the result is described as Eq.(4).

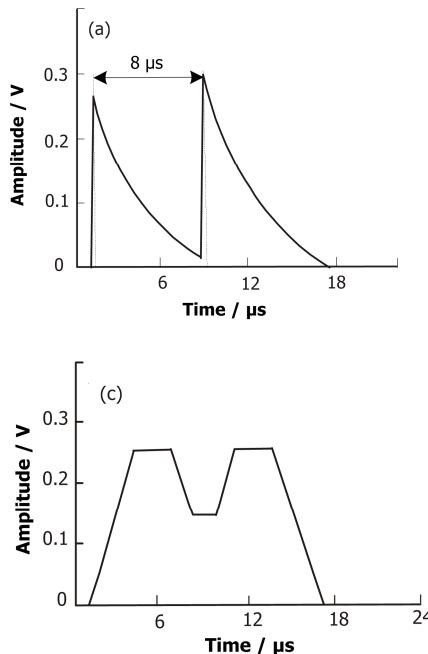
$$H(e^{j\omega}) = \frac{(1-e^{-j\omega t_a} - e^{-j\omega t_b} + e^{-j\omega t_c})(j\omega + 1/\tau)}{(j\omega)^2 t_a} \quad (4)$$

Modulus of Eq.(4) is described as Eq.(5)<sup>[13]</sup>.

$$|H(e^{j\omega})| = \frac{4 \times \sqrt{\omega^2 + 1/\tau^2}}{\omega^2 t_a} \left| \sin\left(\frac{\omega t_a}{2}\right) \sin\left(\frac{\omega t_b}{2}\right) \right| \quad (5)$$

where the parameters are the same as Eq.(1).

Eq.(5) includes two factors, the former is a attenuation factor, with the increase of frequency and the approximate inverse decreased; the latter is a oscillating factor, the oscillation frequency depends on the trapezoidal parameters (rise time,  $t_a$ ). Let  $T_s=25$  ns,  $t_a=1$   $\mu$ s,  $t_b=2$   $\mu$ s,  $\tau=50$   $\mu$ s, the amplitude frequency



response curve of trapezoidal shaping filter is shown in Fig.2.

From Fig.2, when  $t_a$  is greater, the oscillation period is shorter, the low frequency components increase relatively, the better inhibitory effects on high frequency noise. Considering only the noise characteristics, the greater the rise time, the better precision of trapezoidal shaper, it is more conducive to subsequent extraction pulse amplitude information.

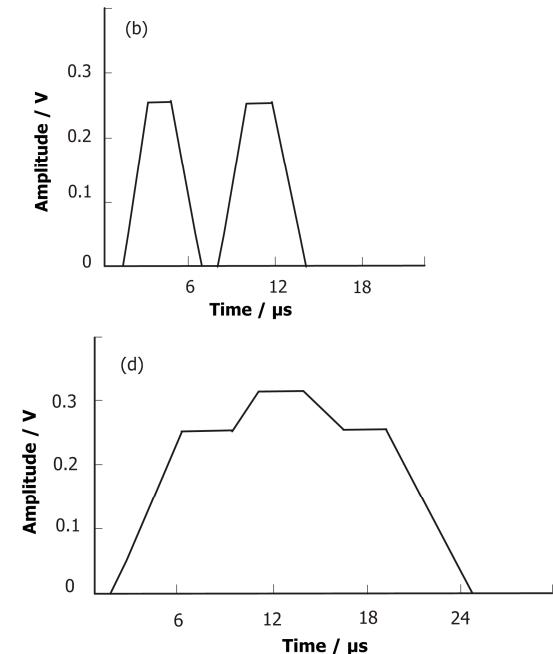


Fig.3 Pulse pile-up of different trapezoidal shaping parameters.

### 3.2 Ballistic deficit characteristics of trapezoidal shaper

In fact, the charge collection time ( $t_d$ ) of detector is greater than 0, and it changes with different pulse, so the ballistic deficit is generated by amplitude extraction in analog filter, it will result in the worse energy resolution of the system. It is shown that it is necessary to study the dependence between shaping process and shaping time parameters. If the current of detector is a constant value and the total charge is  $Q$  in the time  $T_D$ , the feedback capacitor is  $C_f$  in charge sensitive preamplifier. The convolution operation of the input current signal and the pulse response function of preamplifier are accomplished, the voltage waveform of the preamplifier is described as Eq.(6)<sup>[8]</sup>

$$V(t) = \begin{cases} \frac{Q\tau}{C_f t_d} (1 - e^{-t/\tau}) & t < t_d \\ \frac{Q\tau}{C_f t_d} (e^{-t_d/\tau} - 1) e^{-t/\tau} & t \geq t_d \end{cases} \quad (6)$$

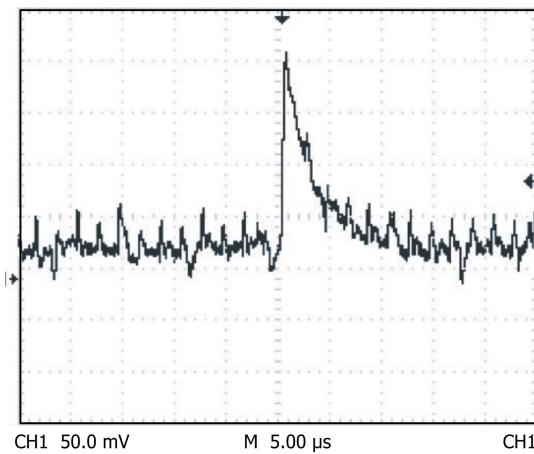
As described in Eq.(7), the signal is sampled and completed differential treatment with  $1 - e^{-T_s/\tau} z^{-1}$ .

$$V'(n) = \frac{Q\tau}{C_f t_d} \sum_{j=1}^{n_d} (1 - e^{-T_s/\tau}) \delta(n - j) \quad (7)$$

where the parameter  $n_d = t_d/T_s$ , The trapezoid is acquired by subsequent processing, which is overlapped the  $n_d$  spacing of 1 and amplitude of  $V(n)$ , when  $n_b - n_a \geq n_d$ , the parameters of  $n_b$  and  $n_a$  are such as Eq.(2), at least one of the  $n_d$  trapezoidal flat is overlapped, the amplitude is described as Eq.(8).

$$V = \sum_{j=1}^{n_d} V'(j) = \frac{Q\tau}{C_f T_s} (1 - e^{-T_s/\tau}) \quad (8)$$

In the above discussion, as long as the trapezoidal flat-top width ( $D$ ) is not less than the maximum charge collection time, the pulse amplitude is proportional to the ideal output amplitude ( $Q/C_f$ ), and has nothing to do with the  $T_D$ . There is no ballistic deficit. Considering only the ballistic deficit characteristics of trapezoidal shape, trapezoidal flat-top width should be larger, in order to eliminate the effect on the pulse amplitude extraction, then it will improve the energy resolution of the system.



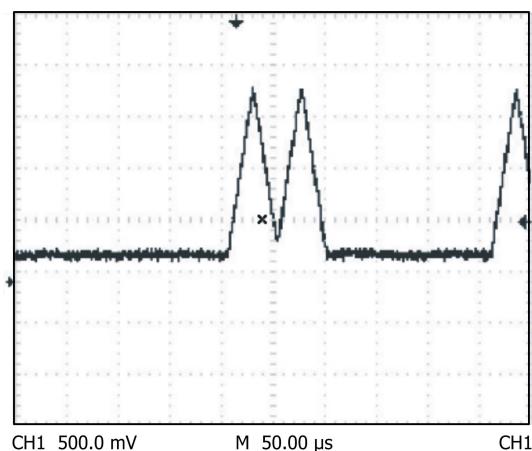
**Fig.4** Nuclear pulse signal before ADC sampling.

### 3.3 Pulse pile-up characteristics of trapezoidal shaper

The pulse pile-up directly affects not only the effective extraction of pulse amplitude, but also the energy resolution and pulse counting rate. The choice of trapezoidal shaping parameters directly affects the pulse pile-up. The results are shown in Fig.3.

From Fig.3, the Fig3(a) shows the original nuclear signal, the pulse interval time is 8  $\mu$ s, Fig.3(b) shows that the ladder of rise time and the flat-top width is 5  $\mu$ s, there is no pulse pile-up, and it can accurately extract the pulse amplitude information. Fig.3(c) shows that the ladder of rise time and the 7- $\mu$ s flat-top width, there is partial pulse pile-up, but also can accurately extract the pulse amplitude information. Fig.3(d) shows that the ladder of rise time and the flat-top width is 10  $\mu$ s, there is completely pulse pile-up, and it is unable to accurately extract the pulse amplitude information. Considering only the pulse pile-up characteristics, which is better when the

shaping parameters are smaller, as discussed above, as long as the interval is not less than the ladder of the rise time and flat-top width, the shaped pulse does not affect the amplitude extraction.

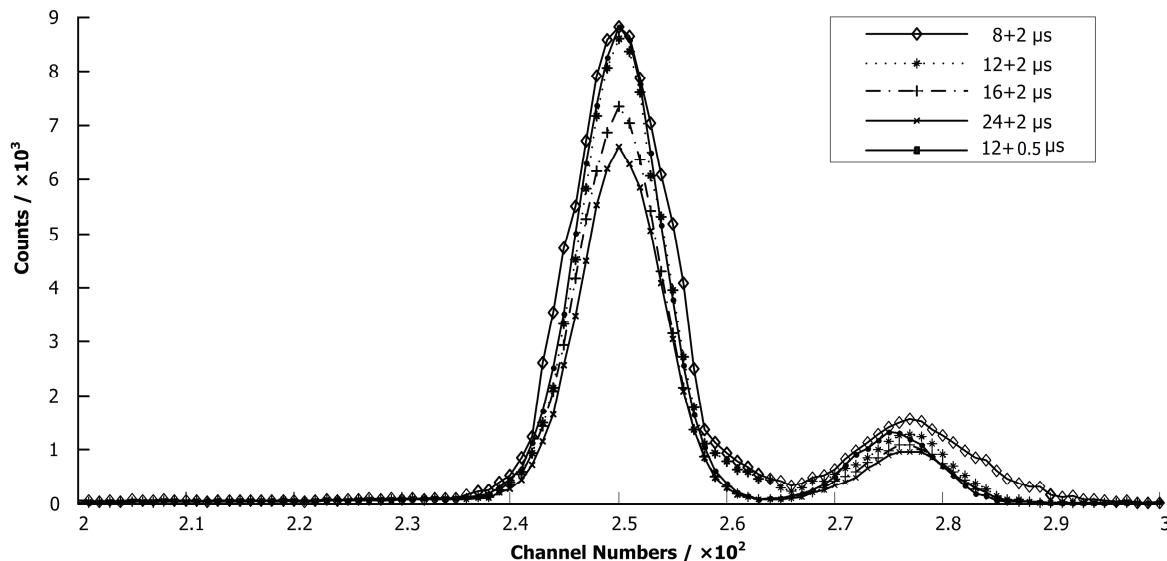


**Fig.5** Trapezoidal shaper of FPGA.

Based on the above discussion, the choice of trapezoidal shaping parameters needs to consider the ballistic deficit, noise, pulse pile-up characteristics and amplitude extraction in digital nuclear spectrometer system. The rise time is longer, the noise filtering performance is better and the energy resolution is higher, and the shaping time is shorter, the probability of pulse pile-up is smaller, the pulse counting rate performance is better, so the optimum filtering parameters are similar to triangle shaper (the longer trapezoidal rise time and the shorter flat-top width).

## 4 Experiment analysis

The Moxtek company's Si-PIN detector of XPIN-XT type and 50 kV X light tube of MAGNUM series in American are used to measure a steel ruler, the voltage of light tube is 20 kV, the current is 2 mA, the rise time and the fall time of nuclear pulse signal of the front ADC are 50 ns and 3.3  $\mu$ s respectively, the result is shown in Fig.4. The pulse sampling point sequence which comes from the ADC (AD9224, 12 Bit, 40 MHz) is shaped into trapezoid in FPGA, the trapezoid is transformed by DAC and sampled by oscilloscope, it is similar to a triangle, which shaping parameters are respective  $t_a=12 \mu$ s,  $D=0.5 \mu$ s (Fig.5).



**Fig.6** Energy spectra of different shaping parameters.

**Table 1** Performance comparison of different shaping parameters

Shaping parameter rise (Time+Width) / $\mu$ s	Energy resolution / eV	Pulse counting rate
8+2	210	8998
12+2	192	8603
16+2	185	7352
24+2	168	6494
12+0.5	195	8877

The energy spectrum is obtained by the same collection data, which is processed in different trapezoidal shaping parameters and pile-up pulse is directly dropped out. The energy spectra are shown in Fig.6, in order to show more clearly, only to show one part of channel numbers, the energy resolution and pulse counting rates of the spectrum are shown in Table 1.

From Table 1, the rise time and flat-top width is increased from 10  $\mu$ s to 26  $\mu$ s, the energy resolution of the system is increased from 210 eV to 168 eV, but the pulse counting rate is reduced from 8998 to 6494. When the shaping time of trapezoid is longer, the energy resolution is higher and the pulse counting rate is lower.

## 5 Conclusion

Trapezoidal shaping method has been widely applied to pulse amplitude extraction and analysis in digital nuclear spectrometer system, the choice of shaping parameters needs to give full consideration to the characteristics of noise, ballistic deficit compensation

and pulse pile-up. Reasonably selecting trapezoidal shaping parameters can improve the energy resolution and pulse counting rate. The testing results show that the trapezoidal shaping parameters should be chosen as long as possible to increase the rise time, in order to enhance the filtering performance for better energy resolution of digital nuclear spectrometer system. The flat-top width should decrease to reduce the probability of pulse pile-up, so the best shaping scheme and the optimal shaping parameters are similar to triangular shaper. Thanks to the correct method in related Ref.[13] which is used to correct the counting rate effectively by the longer shaping time.

## References

- 1 Koeman H. Filtering of signals obtained from semi-conductor radiation detectors. The Netherlands, Katholieke Universiteit Nijmegen, 1973.
- 2 Koeman H. Nucl Instrum Methods, 1975, **123**: 169–180.
- 3 Koeman H. Nucl Instrum Methods, 1975, **123**: 181–187.
- 4 Simoes J B, Correia C M B A. Nucl Instrum Meth A, 1999, **422**: 405–410.
- 5 Radeka V. Nucl Instrum Methods, 1972, **99**: 525–539.

6 Chen S G. Design and realization of the Gaussian shaping filtering in digital nuclear instrument system. Chengdu: Sichuan University, 2005.

7 Yang B, Yan Y J, Zhou J L. Nucl Tech, 2010, **33**: 818–823.

8 Xiao W Y, Wei Y X, Ai X Y. J Tsinghua Univ Sci Technol, 2005, **45**: 810–812.

9 Zhou Q H, Zhang R Y, Li T H. J Sichuan Univ Nat Sci, 2007, **44**: 111–114.

10 Zhou Q H, Zhang R Y, Li T H. Nucl Electron Technol, 2008, **28**: 85–88.

11 Fang Z L, Wang J H, Meng Y, *et al.* Proceedings of the 10th National Conference on Nuclear Instrument and Its Application & the 5th Nuclear Reactor Instrument Conference, 2009, 324–329.

12 Cosimo Imperiale, Alessio Imperiale. Measurement: J Int Meas Confederation, 2001, **30**: 49–73.

13 Zhang H Q, Li L, Wu H X, *et al.* Nucl Tech, 2013, **36**: 020202.

14 Zhang H Q, Wu H X, Tang B, *et al.* J China Technol Nat Sci, 2011, **35**: 67–70.